The most elementary conceivable quantum structure, referred to as the protyposis, allows for a new approach to
interaction, in general, and the shape of concrete gauge groups, in particular.

Here it is essential to realize that the conception of interaction is a manifestation of the dynamical layering
structure, that is, a hybrid from the region where quantum theory and classical physics are overlapping.

As is well known, the four fundamental interactions in physics can be formulated as local gauge theories.
Here the electromagnetic and the weak interactions are associated with force quanta that can appear as real
objects in space-time. For the corresponding gauge groups a relationship to the space-time concept of the
protyposis can be established.

The quanta of the strong interaction, by contrast, cannot exist as free objects. Here and, moreover, in
gravitation many mathematical problems are open. A gauge-theoretical formulation needs to be founded in a
different way.

1. Introduction: Does interaction actually belong to quantum theory?

This question sounds like a provocation and that, to some extent, is intended. As is well
known, contemporary physics appears to allow for a very successful treatment of the four
basic interactions. Each of them can be formulated as a local gauge theory. The large public
echo to the discovery of the Higgs particle has rightly called attention to the fact that the

Simplest Quantum Structures and the Foundation of Interaction

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Subject index: Interaction, quantum theory, quantum structures, gauge groups, electroweak
interaction, cosmic space, dark matter, dark energy, qubits

Abstract:

The final version appeared in:
concept of spontaneously broken local gauge symmetry recovers essential aspects of reality in the case of the electroweak interaction.

However, there are hints that some problems are still waiting for a solution. Normally, local gauge groups are compact. Nevertheless, recent developments in general relativity are based on non-compact gauge groups, for example, the SL(2, C) group, or the group of translations, or the full Poincaré group. The aim here is to quantize the theory, which, however, has not been accomplished so far according to the judgement of the experts.

The quark-gluon interaction described by the SU(3)-color-gauge theory does not allow for a representation of its quanta as free particles in space and time. Even if there is the mathematical structure of a local gauge theory, the question remains as to the physical meaning of such a structure if its subjects can not be realised in space and time. The latter difficulty does not apply to the electroweak interaction. However, up to now it is not clear whether the gauge groups must be guessed from experience or whether there are some deeper connections. The present paper will investigate this question.

Since the age of Newton classical physics evolved along the problem of describing interaction between objects, which are encountered not only in astronomy, but also on earth. Interacting objects are treated by assigning to each object its position and velocity or momentum coordinates, and then specifying the forces between them. Objects are understood as composite if the action of forces may result in an essential change, e.g., fragmentation. Objects are considered non-composite if only their motion changes under the action of forces, but not the objects themselves.

Historically, the first structure of quantum theory, i.e., quantum mechanics, was developed in order to describe interaction, namely, the interaction between electrons and the atomic nucleus. Later on, nuclear physics described the interaction between protons and neutrons in the nuclei in terms of mesons. With the advent of quantum field theory the quantum-
theoretical descriptions of force fields became possible. The treatment of interactions between force fields and matter fields is routine in current quantum physics.

This seems to indicate that interaction is a well-established basic concept in quantum theory.

However, a closer inspection suggests that things are not that simple as they appear at first sight.

*The problem is that “interaction” is a concept which cannot be reconciled with the mathematical structure of quantum theory.*

Admittedly, this sounds disconcerting, if not to say strange. However, a somewhat deeper analysis of the mathematical structure of quantum theory will readily offer an explanation.

Why should interaction not fit into quantum theory?

This apparently strange finding is a consequence of the basic structures of quantum theory.

\[ \mathbf{H} = \mathbf{H}_1 \otimes \mathbf{H}_2 \]  

According to this structure, in any interaction the overall system is represented mathematically by the tensor product of the state spaces of the subsystems.

The states of the overall system that can meaningfully be seen as “consisting” of the subsystems are pure product states of the original subsystems. These states, however, only form a set of measure zero in the state space of the overall system. For example, the coordinate lines in the (x,y)-plane are null sets. In the mathematical treatment, as a rule, null sets can almost always be ignored. This means that exactly those states can be ignored as being irrelevant where speaking of an interaction between two objects, would, in fact, be justified.

For all other states (this means for almost all states) the term “entanglement” ("Verschränkung") applies. The German term “Verschränkung” (originally introduced by
Schrödinger\textsuperscript{1}, deriving from carpentry, is unfortunate as it is suited to prevent any understanding. Two planks that are “verschränkt” or even two ropes that are “entangled” remain two planks and two ropes, respectively. By contrast, two entangled particles constitute a new quantum system that is a whole, an entity without parts. It can be decomposed into the original parts – or into something totally different – but, as a whole, it does not “consist” of the parts before a decomposition happens.

The underlying mathematical structure means that, strictly speaking, the concept of interacting subsystems misses the essence of the quantum theoretical description of reality. The idea of interaction presupposes the lasting existence of separate objects for which interaction is mediated by forces. This, however, is inconsistent with the wholeness without parts which, in quantum theory, is reflected by the tensor product.

In the previous course of physics, this problem of interaction has been concealed by an explicit or mostly only implicit use of superselection rules. In case of a superselection rule, the superposition of states associated with different objects exposed to an interaction is excluded. Therefore their individual existence can still be supposed. As long as quantum particles or quantum fields, maintaining their identity even in case of interaction, are taken as the basis of quantum-theoretical considerations, the conflict between pure quantum theory and interaction does not arise in its full rigor.

However, if one tries to reduce physics to the really simplest quantum structures, the problem of the actual non-existence of interaction emerges with utmost clarity.

For the treatment of interaction we therefore need to take recourse to the “dynamical layering-structure” of classical and quantum physics. [1] Herewith not only the problem of measurement will become understandable [2], but it also makes us aware that for human beings the henadic description of the world by quantum theory must be supplemented with

\textsuperscript{1} Erwin Schrödinger was so annoyed about the „damned quantum jumping“ that after the mid 1930\textsuperscript{ies} he work predominantly and very successfully in the field of general relativity.
the world view of classical physics, in which the existence of separated objects is a natural ingredient.

Before dealing with this problem we have to clarify what a “simplest quantum structure” could possibly be. Here also the notion of an “object” must be examined, as well as the ramifications of such an analysis for the description of interaction.

**2. What are the simplest quantum structures?**

At present, quantum field theory is the basis for most of fundamental research in physics. Relativistic quantum field theory, combining quantum theory and special relativity, has emerged as an excellent description of localized elementary processes in space and time – irrespective of all of its considerable mathematical inconsistencies. The necessity of setting infinity to zero in a Lorentz-invariant way in the process of renormalization indicates that there is need for further clarification.

Treating problems within the frame of special relativity means amongst others that the experiments are described as occurring not in the real cosmic space but in its tangential space – the Minkowski space, which is justified due to the extremely small curvature of the cosmic space. In physics, apart from professional cosmology, the Minkowski space is seen as the “quasi natural frame” for physical phenomena – mostly without much reflection, though certainly not without some justification. In elementary particle physics, for a long time any connection to cosmology was denied because the opinion prevailed that the so-called “microphysics” has nothing to do with “macrophysics”! As I personally witnessed 30 years ago, serious hints at relations between quantum theory and cosmology were declared unfounded and ignored. However, things have changed in the last decade. “Astro-particle-physics” has become an important field in physics.

A mathematical impeccable definition of an object, i.e. a relativistic particle, can be given in Minkowski space. Such a particle can be moved in space and time without changing its
essential aspects. Interaction between objects is mediated by forces. According to the
relativistic treatment, the forces can ultimately be represented by quantum fields.

As is well known, quantum fields have uncountably infinite non-equivalent
representations. The simplest of them is the Fock representation. Here the state space of the
quantum field $\mathbf{H} (\Phi)$ is the direct sum of tensor products of the one-particle statespaces $\mathbf{H}_1$ of
the field quanta:

$$\mathbf{H} (\Phi) = \sum_{n=0}^{\infty} \mathbf{H}_1 \otimes \cdots \otimes \mathbf{H}_n$$

(2)

As first proposed by Einstein, certain phenomena of the interaction between light and
matter can be understood only if particle properties are attributed to light, that is, light being
constituted by photons. Einstein’s photon nature of light can be formulated mathematically in
terms of the Fock representation. In a similar way, any quantum field can be understood as a
potentially infinite set of its field quanta, i.e., quantum particles.

*As to the search for the simplest quantum structures this makes apparent that quantum
particles are essentially simpler than quantum fields.*

A quantum particle is a quantum theoretical description of what one would call, in
everyday language, an object without inner degrees of freedom. Here a clear mathematical
representation was established by Eugene Wigner [3].

An elementary object is characterized by the fact that it can be moved in space and time
such that its state changes but not the object itself. The group of all possible motions was
called by Wigner the inhomogeneous Lorentz group. It comprises three rotations in space,
three “hyperbolic rotations” between space and time coordinates – the so called boosts –
which together give rise to the SO(3, 1) Lorentz group. The Lorentz group is locally
equivalent to the SL(2, C) group. Furthermore, there are four translations. The resulting
group of ten generators is referred to as the Poincaré group. As already mentioned, an
elementary object is an entity without inner degrees of freedom. Accordingly, its states form
an irreducible representation of the Poincaré group. With the exception of the vacuum state, each of these representations spans an infinite-dimensional Hilbert space.

Here the particle type is completely determined by the invariants of the representation. These invariants are the values of mass and spin.

A real object is either massless, like a photon, or it has a positive mass. The spin can have integer values, 0, 1, 2, …, in which case the particles are bosons, the quanta of forces. Or the spin has half-integer values, 1/2, 3/2, …, then the particles are fermions, the quanta of matter. The state with vanishing mass and spin, and also with vanishing energy and momentum is called the vacuum.

*The infinite dimension of the Hilbert space of a quantum particle is an indication that an elementary object, i.e., a “particle”, does in no way constitute a simplest quantum structure yet.*

In classical physics, the state spaces of interacting objects are combined in an additive way, the total dimension being the sum of the dimensions of the object spaces. In classical mechanics, one particle is described in the six-dimensional position and velocity space, so that a 12-dimensional space is assigned to two particles, and an 18-dimensional space to three particles. The individuality of each object is conserved by this structure.

In contrast to classical physics, quantum theory combines – as already mentioned – interacting objects in a multiplicative way, that is, by forming the tensor product of the state spaces. Here, the dimension of the state space of the whole system is given by the product of the dimensions of the state spaces of the parts. In this way higher dimensional state spaces can be formed of lower dimensional ones. (Quantum structures with one-dimensional state space are not conceivable.) In such a multiplicative composition, the individuality of the original objects is no longer preserved.
If one looks for the actually simplest quantum structures, one must expect that they have only a two-dimensional state space.

For a structure with a two-dimensional state space the idea of a further decomposition into something even simpler is absurd. On the other hand, the state spaces for more complex entities up to quantum particles and quantum fields can be generated in terms of a sufficiently large or infinite number of those elementary quantum structures.

Using infinite tensor products of two-dimensional spaces, one can generate even all those state spaces necessary for the representation of quantum fields beyond the Fock representation. Since the work of John von Neumann [4] it is known that these infinite tensor products generate superselection rules, being a characteristic of the various non-equivalent representations of quantum fields.

The significance of elementary quantum structures with two-dimensional state spaces is that, on the one hand, they are actually the simplest structures conceivable in quantum physics, and, on the other hand, allow one to generate all the systems and structures which are the subject of quantum theory.

However, as is absolutely clear from the outset, those elementary structures must neither be perceived as objects – which would imply infinite-dimensional space states because of localizability in space and time – nor as quantum fields – having vastly more degrees of freedom then an object or a quantum particle, and a state space of uncountably infinite dimension.

Which intuitive ideas can be developed for such really elementary structures?

3. Intuitive ideas for the most elementary quantum structures

Two-dimensional state spaces are well known in quantum theory for a long time. One example is the spin, e.g., of an electron. The spin states span a two-dimensional space;
however, a spin is always linked to a real existing particle. A “spin without a particle” would be a strange conception.

Another physical model with a two-dimensional state space is the quantum bit.

Since the mid 1950ies, Carl Friedrich v. Weizsäcker had pioneered the idea that it should be possible to construct physics on the basis of quantized binary alternatives.[5] Weizsäcker had called them “Uralternativen” or “Ure” (plural form of Ur), today they are referred to as qubits. Also David Finkelstein [6] and later, among others, Archibald Wheeler advanced such ideas, for which Wheeler coined the memorable slogan "It from Bit".[7]

However, given the concept of urs or qubits the problem is how to establish the physical meaning of such structures.

Weizsäcker’s starting point was quantum logic, the temporal logic of decidable alternatives. His urs were understood as quantum information and, thus, in a broader sense as “information”. In the work of Weizsäcker the ur connotes with potential knowledge and therefore with meaningful information.

To this interpretation as “information” and the thereby established connection to knowledge objections were raised by a number of parties. Above all, problems arise with the herewith implied specific role of an observer as the bearer of knowledge. As long as the observer is assumed to be quasi outside of physics, there is a scientific problem, because after all even the observer should be amenable to a scientific and evolutionary description.

Eventually, an observer with the capability of consciousness has not always existed. A “conscious observer” evolved from precursors without consciousness, and these life-forms evolved from some inanimate matter. In view of such an evolutionary development, science is challenged to describe and explain the course of this evolution with all of its particular transitions.
Of course, the description of physical phenomena is inevitably linked to a “describer” – in which form however. Still, in a scientific approach one has to insist that the special role of a human observer must not be overemphasized. It is one of the tacit premises of science that human beings enter by birth a reality which endures after their demise. Occasionally, one can find statements implying that according to quantum theory “we create our reality”, clearly a rather misleading perception. It is essential to overcome the still existing view that the “observer” should be beyond the physical description.

*Therefore it is essential here to further extend the scientific abstraction beyond Weizsäcker’s concept, abstracting not only from emitter and receiver but also from knowledge and the notion of meaning.*

This is also necessary because “meaning”, being a characteristic of information – in the way this word is used in everyday language – can hardly be dealt with in science due to the significant subjective aspects of meaning which is at odds with an objectivity-orientated science. In fact, information is almost ever thought as being something “meaningful” forwarded by an emitter to a receiver.

When we are about to consider the “most elementary and simplest quantum systems”, it is useful to recall that in everyday life “information” is always linked to an energetic or material carrier, e.g. the photons of light, or air in the case of sound, or the paper of a book. Obviously, all those carriers of information, having infinite dimensional state spaces, are by no means something “simple”.

*With regard to our most elementary quantum structure with its two-dimensional state space, one has to abandon any thought of an emitter and receiver, a carrier, and, above all, specific meaning. One has to abstract from all those terms.*
A premature acceptance of the most elementary quantum structure as “quantum information” can only lead to inappropriate conceptions, which also applies to an interpretation as “spin”.

Because the most elementary quantum structure is neither a localisable particle nor a quantum field, nor quantum information in the usual sense, nor a spin, it is appropriate to introduce a new notion here. A new notion should be instrumental in avoiding obvious misconceptions from the outset.

Following a suggestion by Roland Schüssler, the late classical philologist at the Frankfurt University, the notion “Protyposis” has been introduced.[8] In the Greek-German dictionary of Wilhelm Pape, the meaning of "προτύπωσις" is "das Vorbilden", perhaps “prefiguration” or “foreshadowing” in English. As far as we can tell, the notion Protyposis, when heard for the first time, does not induce any specific, let alone some inappropriate connotations. On the other hand, this conception comprises the potential of the structure it denotes, the potential to unfold all of the reality encompassing us, that is, both matter and consciousness.

Primarily, the Protyposis is non-local in space and time, and lacking any special meaning.

If, however, one prefers to associate the Protyposis with quantum information in a very broad sense, it must be conceived as abstract and free of special meaning and without any recourse to an emitter and receiver.

Because the quanta of the Protyposis, supposing their existence, cannot like objects be “here and now”, they must be perceived as being “always and everywhere”.

The concept of quantum structures that do not represent particles or quantum fields, is utmost abstract and alien to our imagination, transcending by far the conventional framework of physical notions and concepts. While the protyposis appears extended like a quantum field, it does not have the infinitely many degrees of freedom.
Because the qubits of the Protyposis cannot be localized, they must be understood as being something “cosmic”. For the sake of illustration, one might view the qubit as a “fundamental oscillatory mode” of the cosmic space.

The sine function may serve as an illustrative analogy. It can be seen as a fundamental oscillation of the unit circle. In contrast to a field in which every point in space may be mapped to an individual value, the sine function is completely determined on the unit circle by specifying one point, e.g., the position of its maximum.

![Sine function graph](image)

**Fig. 1: Sine and the 10001st power of sine between 0 and 2 \( \pi \)**

Taking the sine as metaphor for a quantum bit of the protyposis, then many qubits will correspond to a corresponding power of the sine. If they are all in the same state – if for every factor the sine maximum is at the same position – then the product results in a strongly localized function.

*The non-locality of the qubits differs fundamentally from the extended structure of a quantum field.*
Obviously, quantum theory suggests that something localised can be constructed from a manifold of something extended – an, at first glance, completely counterintuitive idea. This insight is a radical antithesis to the antique notion of atoms which physics has adopted for more than two and a half millennia. At the core of the atomic hypothesis is the belief that the simple is to be found in the small, necessitating to postulate ever smaller “particles”. By contrast, quantum theory allows us to understand that something simple may be extended and something small can be complex.

The proposed connection between protyposis and the cosmic space is not just an arbitrary assumption, as will be outlined in the next chapter. It will be seen that the increase of the number of quantum bits generates an effect to be interpreted as the expansion of the cosmic space. Given sufficiently many quantum bits, material objects can originate in space, and, later in the cosmic evolution, living beings can form – even living beings in which the protyposis can appear in the form of consciousness. Ultimately, the conception of protyposis will allow one to include the observer into the frame of physics, not only his brain but also his conscious mind.

4. The origin of space from quantum theory

All our experiences, all experiments, and observations happen in the three-dimensional space we live in, and in time. However, ever since the mathematicians proceeded beyond the ancient Greek geometry to higher-dimensional spaces, the three-dimensionality of physical space is in need of rationalization.

Presently, one can often read that, actually, space is eleven-dimensional and the four-dimensional space-time is only the result of compacting the other dimensions. When, moreover, it is insinuated that the eleven dimensions are essentially self-evident, rendering a further justification dispensable, one should be assured that some further reflection in natural philosophy is advisable.
Carl Friedrich v. Weizsäcker was the first to postulate that the three-dimensionality of the physical or position space is a necessary consequence of quantum theory. [9] How can this postulate maintained today? Would it be possible, on the basis of the protyposis, to go beyond the explanation of the three dimensions and establish new insights about space and its structure?

The states of a quantum bit of the protyposis have a symmetry group for which the absolute value of the scalar product is invariant. This is essentially the SU(2) group, but comprises also U(1) and the complex conjugation.

Disregarding for the moment complex conjugation and U(1), the states of the qubit can be represented by the group elements of the SU(2) group. The action of the SU(2) group elements on a single qubit state generates all qubit states. A two-dimensional representation of the SU(2) group corresponds to the states of our most elementary quantum structure. All composite structures with higher-dimensional state spaces arise from tensor products of such two-dimensional representations.

In the theory of compact groups, an important theorem states that all irreducible representations of the group can be represented in the Hilbert space of the square-integrable complex-valued functions defined over the group itself. As a homogeneous space, the SU(2) group is a $S^3$ manifold, the three-dimensional “surface” of a four-dimensional sphere. As a consequence of this theorem, any structure generated by the protyposis can be represented, together with all its states, by functions over this $S^3$ manifold. This suggests to view this space as a mathematical model for the position space, i.e., the real cosmic space. As a consequence, all what can be formed of the protyposis will appear as being realized in a three-dimensional space.
The two-dimensional representation of a single qubit consists of functions which have only one null-plane on the $S^3$. They divide the $S^3$ space into „two halves“. The wave length of these functions corresponds to the diameter of the $S^3$ space. In analogy to the sin-function – where $\sin(nx)$ has $2n$ zero-points – the functions of the representations $(2n+1)D_n$ divide the $S^3$ by $n$ null-planes, therefore the wavelength is of order $R/n$ ($R$ is the radius of the $S^3$). In the tensor product $(\otimes_{l=2})^N$ of $N$ of these two-dimensional representations there are also

$$
(\otimes_{l=2})^N = \bigoplus_{j=0}^{[N/2]} N!(N+1-2j) \left( \frac{2|N/2|\cdots 2j+1}{(N+1-j)!j!} D_{|N/2|-j} \right)
$$

functions with substantially shorter wave lengths. The corresponding Clebsch-Gordan series for the decomposition into irreducible representations is of the form (we define $|N/2|=k$ for $N=2k$ or $N=2k+1$).

The multiplicity $f(j)$ for the respective irreducible representations with a state space of dimension $(2|N/2|-2j+1)$ follows from the reduction of the tensor product in the Clebsch-Gordan series [10]:

$$
f(j) = \frac{N!(N+1-2j)}{(N+1-j)!j!}
$$

The multiplicity factors grow almost linearly from $j=N/2$, where $f(N/2) = O(2^N N^{3/2})$ for the largest wavelength $D_0$ or $D_{1/2}$, to a maximum at $j=(1/2)(N - \sqrt{N})$ for the representation $D_{(\sqrt{N})}$, where

$$
f[(1/2)(N - \sqrt{N})] = O(2N N^l) = (\sqrt{N}) \cdot O(2N N^{3/2}) = (\sqrt{N}) \cdot f(N/2)
$$

Beyond that, there is an exponential decrease towards the value 1. For large $N$ the maximum is very sharp, and functions with a wavelength smaller than $2\sqrt{N}$ have a small weight and can
be ignored. Based on this group-theoretical definition, a smallest length on the $S^3$ space can be established, which will be identified with the Planck-length $\lambda_0$.

### 5. From the most elementary quantum structures and the space herewith established towards a concept of time and energy

According to the group-theoretical considerations given above, a rise in the number of qubits can be associated with an increasing extension of the cosmic space. The more qubits are available, the finer a division of the cosmic space is possible. Using the Planck length $\lambda_0$, the smallest physically feasible length, as a constant length unit, it follows that the cosmic curvature radius $R$ increases with the root of the number $N$ of the qubits:

$$R = \lambda_0 \sqrt{N}$$

Once a length has been defined, the postulate of a universal velocity allows for a definition of time.

As we know from physical experience, there is a universal and invariant velocity, namely the velocity of light in vacuum, and it is natural to use this as a reference. With

$$R = c t$$

an expression for the temporal evolution of the expansion of the cosmic space is obtained, which, moreover, allows one to determine the temporal evolution of the number of qubits in cosmic space:

$$N = R^2/\lambda_0^2 = (c t)^2/\lambda_0^2$$

According to a basic postulate of quantum theory, the extension $\lambda$ of a quantum system is inversely proportional to its energy $E$.

If the qubit can be understood as a fundamental oscillation of the cosmic space, its extension $\lambda$ is of the order $R$. Using Planck’s formula,
where $h$ is Planck’s constant, the energy of a qubit is given by

$$E_q = h c / R$$

As a consequence of this assumption, the total energy of all $N$ qubits is

$$U = N E_q = N h c / R = N h c / \lambda_0 \sqrt{N} = \sqrt{N} h c / \lambda_0 = R h c / \lambda_0^2$$

The volume of the $S^3$ space is $2 \pi^2 R^3$, so that the energy density $\mu$ reads

$$\mu = U / 2\pi^2 R^3 = (R h c / \lambda_0^2) / 2\pi^2 R^3 = \sqrt{N} h c / \lambda_0 2\pi^2 (\lambda_0 \sqrt{N})^3 = h c / 2\pi^2 \lambda_0^4 N$$

or

$$\mu = h c / \lambda_0^2 2 \pi^2 R^2$$

In the cosmic expansion, the energy density and also the total energy as given above do not remain constant. In view of a changing energy, we may, nevertheless, require at least the validity of the first law of thermodynamics:

$$dU + p dV = 0.$$  

then

$$dR \frac{h c}{\lambda_0^2} + p 2\pi^2 3 R^2 dR = 0$$

or

$$p = -\frac{h c}{\lambda_0^2} p 2\pi^2 3 R^2 = -\frac{\mu}{3}$$

Thus, using quantum-theoretical arguments and adopting a thermodynamical postulate, we have arrived at a cosmological model, which provides for a natural explanation of the so-called “dark energy”, as will be discussed in the following.

**5.1 What is “dark energy”?**

Pressure is usually assumed to be a positive physical entity. In cosmology, pressure has been neglected for a long time. While the possibility of a negative pressure has not even been considered, positive pressure plays in fact only a minor role, since the cosmic space is nearly
empty, so that no gas pressure can form, and the pressure of electromagnetic radiation is negligible. However, pressure is a constituent of the energy-momentum tensor and, as such, has a gravity effect. Only a negative pressure can counteract the gravitational attraction. In cosmological models with a sufficiently large content of normal gravitating matter and non-negative pressure there is no permanent expansion of space, rather the expansion will come to a halt and turn into a contraction. According to modern astronomical observations, such behaviour can be excluded with sufficient certainty. In particular, the observations indicate that an end of the expansion cannot be expected. Less certain is the experimental situation with regard to an accelerated expansion of space, being presently the preferred hypothesis.

To explain the obviously missing deceleration in the conventional cosmological models, the existence of an ominous “dark energy” has been postulated. By definition, dark energy should counterbalance the gravitational effect of the matter content of the cosmos by an expanding action.

*Within the protyposis cosmology the previously inexplicable conjecture of dark energy becomes a logical consequence of quantum-physical concepts.*

The observed expanding behaviour of the cosmos proves to be simply the result of the negative pressure of the protyposis, being a necessary consequence of the first law of thermodynamics.

The protyposis cosmology solves also the other “cosmological problems” without resorting to further ad hoc-assumptions, such as an unexplained “inflation”. The mysterious smallness of the cosmological term and the so-called horizon problem prove to be logical consequences of the protyposis cosmology.
5.2 Einstein’s equations and quantum cosmology

Supposing that the relations between the structure of space and time and the energy-momentum tensor should also hold for local variations in the cosmic energy density, it becomes obvious that in the description of such fluctuations Einstein’s equations apply. [11]

Local arrangements of the protyposis will satisfy Einstein’s equations. However, this does not mean that the protyposis is formed completely in the shape of elementary objects, i.e., of more or less localizable particles. While the protyposis has not to be “realized” entirely as “particles”, one has to expect gravitational interaction between particle forms of the protyposis and such protyposis structures that cannot be understood as localized objects. Accordingly, some parts of the protyposis may be locally denser in some areas than in other areas of the cosmic space, inducing “dark matter” type gravitational action without the need for postulating some unknown or inexplicable particles.

6. From the most elementary quantum structures to elementary objects, i.e., particles

As already mentioned, elementary objects, particles, have been defined in a mathematically clear way by Wigner. The assumption here is that the Minkowski space is an appropriate model of the physical space-time in which they are described. The protyposis cosmological model tends to the Minkowski space for $N \to \infty$. Also, potentially infinitely many qubits are required for the construction of an irreducible representation of the Poincaré group with its infinite-dimensional state space.

These infinities arise necessarily in the context of a rigorous mathematical modelling. Of course, it is physically inconceivable to accelerate an electron in such a way that its inertia assumes a value as large as that of whole galaxy. However, in order to construct a unitary representation of the Poincaré group, the recourse to such physically unrealistic states in the
mathematical modelling is legitimate and necessary. As a side remark, we note that our mathematical models, however good they may be, must always be seen as mere approximations to reality.

Before turning to interactions between objects, it is necessary to address the elementary objects known in physics, that is, structureless relativistic particles, expediently described in the tangential Minkowski space of the cosmic space-time.

Among the norm-conserving symmetry transformations of the qubits is the complex conjugation. Here a linear representation, as suggested by quantum theory, can be obtained via a duplication of the qubit types. Already in Weizsäcker’s ansatz, urs and anti-urs were introduced. This duplication establishes a four-dimensional state space. Using the qubit- and anti-qubit states as the basis of a Fock representation, it becomes possible to construct, in a “second quantization” procedure, relativistic particles from these quantum bits.

Supposing Bose statistics for the quantum bits, massless objects will result. To generate massive particles, Para-Bose statistics is required. For completeness, let us briefly outline some details. The tri-linear Para-Bose commutation relations are defined as follows:

\[ \frac{1}{2} [\{ a_r, a_s^\dagger \}, a_t] = -\delta_{st} a_r, \]
\[ [\{ a_r, a_s \}, a_t] = [\{ a_r^\dagger, a_s^\dagger \}, a_t] = 0 \] (14)

Likewise, these relations can be written in the form

\[ \frac{1}{2} [a_r, \{ a_s^\dagger, a_t^\dagger \}] = -\delta_{st} a_r^\dagger - \delta_{sr} a_r^\dagger, \]
\[ \frac{1}{2} [a_s^\dagger, \{ a_r, a_t \}] = -\delta_{sr} a_s - \delta_{rt} a_r, \]
\[ \frac{1}{2} [a_t^\dagger, \{ a_r^\dagger, a_s \}] = -\delta_{sr} a_t^\dagger \] (15)

Here (r, s, t = 1, 2, 3, 4)
According to the Green decomposition [12], Para-Bose statistics can be interpreted as

\[ a_s = \sum_{\alpha=1}^{n} b_s^{a\alpha}, \quad a_s^\dagger = \sum_{\alpha=1}^{n} b_s^{\dagger a\alpha} \]

with

\[ [b_r^{\alpha\alpha}, b_s^{\dagger\alpha}] = \delta_{rs}, \]
\[ [b_r^{\alpha\alpha}, b_s^{\alpha\alpha}] = [b_r^{\dagger\alpha\alpha}, b_s^{\dagger\alpha\alpha}] = 0 \]
\[ \{b_r^{\alpha\alpha}, b_s^{\dagger\alpha}\} = \{b_r^{\dagger\alpha\alpha}, b_s^{\alpha\alpha}\} = \{b_r^{\dagger\alpha\alpha}, b_s^{\dagger\alpha}\} = 0 \quad \text{for} \ \alpha \neq \beta \]

applying to different types of bosons which are commuting in case they are of the same type, and anti-commuting in case they are of different types.

Finally, denoting the protyposis vacuum by \(|\Omega\rangle\), the Para-Bose-order by \(p\), where \(p=1\) restores Bose statistics:

\[ a_s a_s^\dagger |\Omega\rangle = \delta_{rs} p |\Omega\rangle \]

The Poincaré group is the ten-parameter group of motions in the Minkowski space \(\mathbb{R}(3,1)\).

It is a semidirect product of the Lorentz group \(SO(3,1)\) and the four translations. The \(SO(3,1)\) group is – as already mentioned – locally equivalent to the \(SL(2,\mathbb{C})\) group. This special linear group in two dimensions would be a general symmetry group for the qubits if the conservation of the norm was no longer a constraint. As Scheibe, Süssmann and v. Weizsäcker conjectured a long time ago, this could be a rationalization of the role of this group in special relativity. [13]

As mentioned above, the norm-conserving symmetry group of a quantum bit of the protyposis comprises, beside the SU(2), also a U(1) group. Whilst the SU(2) describes the space-like aspects of the qubit states, the time-like aspects can be related to the phase transformations of the U(1). If one accepts a cosmology in which the number of qubits is growing, the unitary U(1) description must be changed. This can be achieved by using a complex time parameter rather than a real one. Here space and time will be subject to a
pseudo-euclidean metric which turns into the Minkowski space for an infinite number of qubits, \( N \to \infty \).

The vacuum in Minkowski space, the Lorentz vacuum \(|0\rangle\), is an eigenstate of the Poincaré-group with mass, energy, and spin zero. It can be constructed on the qubit vacuum \(|\Omega\rangle\) as follows:

\[
|0\rangle = \sum_{n=0}^{\infty} \sum_{n_1,n_2=0}^{n} \frac{(-1)^n a_1^\dagger a_3^\dagger + a_2^\dagger a_4^\dagger}{2^n n_1! n_2!} \left( \frac{a_3 a_4^\dagger + a_4 a_3^\dagger}{2} \right)^n |\Omega\rangle
\]

(18)

It shows that the statement "at every point in Minkowski-space there is no particle" corresponds to a potentially infinite amount of information. Besides the qubits (indices 1 and 2), also anti-qubits (indices 3 and 4) are introduced. As mentioned above, this allows for a linear representation of complex conjugation, being a subgroup of the qubit symmetry group. In the Minkowski space vacuum, the annihilation of a qubit is equivalent to the creation of its anti-qubit:

\[
a_1|0\rangle = -a_3^\dagger|0\rangle \quad a_2|0\rangle = -a_4^\dagger|0\rangle \quad a_3|0\rangle = -a_1^\dagger|0\rangle \quad a_4|0\rangle = -a_2^\dagger|0\rangle
\]

(19)

The construction of particle states and Casimir operators quickly become very complicated, and the use of computers is advisable. The following expressions have been generated using mathematica®. Here the computer-adapted notations for the creation and destruction operators are used:

\[
\begin{align*}
a_1^\dagger & \Rightarrow \psi[r] \quad \text{(Erzeuger)} \\
a_i & \Rightarrow \nu[r] \quad \text{(Vernichter)} \\
\{a_1^\dagger, a_3^\dagger\} & \Rightarrow 2f[r,s] \\
\{a_2^\dagger, a_4^\dagger\} & \Rightarrow 2w[r,s] \\
\{a_1^\dagger, a_i\} & \Rightarrow 2d[r,s] \\
|0\rangle & \Rightarrow \text{lvac}
\end{align*}
\]

(20)

The symbol *denotes the multiplication, ** the noncommutative multiplication and I* the multiplication with the imaginary unit.
The generators of the Poincaré group take on the following form:

**Translations:**
\[
P^1 = (-w_{2,3}-f_{3,2}-w_{1,4}-f_{4,1}-d_{1,2}-d_{2,1}-d_{4,3}-d_{3,4})/2
\]
\[
P^2 = I^*(-w_{2,3}+f_{3,2}+w_{1,4}-f_{4,1}-d_{1,2}+d_{2,1}-d_{4,3}+d_{3,4})/2
\]
\[
P^3 = (-w_{1,3}-f_{3,1}+w_{2,4}+f_{4,2}-d_{1,1}+d_{2,2}-d_{3,3}+d_{4,4})/2
\]
\[
P^0 = (-w_{1,3}-f_{3,1}-w_{2,4}-f_{4,2}-d_{1,1}-d_{2,2}-d_{3,3}-d_{4,4})/2
\]

**Boosts:**
\[
M^{10} = I^*(w_{1,4}-f_{4,1}+w_{2,3}-f_{3,2})/2
\]
\[
M^{20} = (w_{1,4}+f_{4,1}-w_{2,3}-f_{3,2})/2
\]
\[
M^{30} = I^*(w_{1,3}-f_{3,1}-w_{2,4}+f_{4,2})/2
\]

**Rotations:**
\[
M^{32} = (d_{2,1}+d_{1,2}-d_{3,4}-d_{4,3})/2
\]
\[
M^{21} = (d_{1,1}-d_{2,2}-d_{3,3}+d_{4,4})/2
\]
\[
M^{31} = I^*(d_{2,1}-d_{1,2}-d_{3,4}+d_{4,3})/2
\]

The \(n\)-fold non-commutative product \(f_{r,s}\), i.e. \(f_{r,s}**f_{r,s}**...**f_{r,s}\), is abbreviated by \(f_{r,s,n}\).

The two Casimir operators of the Poincaré group are the square of the mass
\[
m^2 = (P^0)^2 - (P^1)^2 - (P^2)^2 - (P^3)^2 =
\]
\[
-(d_{1,1}+d_{3,3}+f_{3,1}+f_{4,2}-d_{1,1}**w_{2,4}+d_{1,2}**w_{1,4}+d_{2,1}**d_{1,2}+d_{2,1}**w_{2,3}-d_{2,2}**d_{1,1}-d_{2,2}**w_{1,3}-d_{3,3}**d_{2,2}-d_{3,3}**w_{2,4}+d_{3,4}**d_{1,2}+d_{3,4}**w_{2,3}+d_{4,3}**d_{2,1}+d_{4,3}**d_{3,4}+d_{4,3}**w_{1,4})
-\]
\[
d_{4,4}**d_{1,1}-d_{4,4}**d_{3,3}-d_{4,4}**w_{1,3}-f_{3,1}**d_{2,2}-f_{3,1}**d_{4,4}-f_{3,1}**w_{2,4}+f_{3,2}**d_{1,2}+f_{3,2}**d_{4,3}+f_{3,2}**w_{2,3}+f_{4,1}**d_{2,1}+f_{4,1}**d_{3,4}+f_{4,1}**w_{2,3}+f_{4,2}**d_{1,1}-f_{4,2}**d_{3,3}-f_{4,2}**f_{3,1}-f_{4,2}**f_{3,2}+f_{4,2}**w_{1,4}-f_{4,2}**w_{2,3}+f_{4,2}**w_{2,4}+w_{1,4}**w_{2,3}+w_{1,3}+w_{2,4})
\]
and the Pauli-Lubanski-operator is $W^2 = W_a W^a$ where $W_a = (1/2) v_{abcd} M^{ab} P^c$. Its explicit form has been given in [14]. Here also explicit states of massless and massive particles can be found. As two examples, let us consider the states of a massless fermion and a massive spinless boson at rest.

A massless fermion moving in z-direction, e.g. with momenta $P^0 = P^3 = m$ and $P^i = P^2 = 0$, and helicity $p|2\rangle + (1/2)$, has the form

$$\sum_{p^0 > 0} \frac{(2 p^2 + 1 + p^0 - 1)!}{p^1!} \frac{(-m)^{p^1}}{p^0!} \eta[3, 1, p^1] \eta[1, 1, p^2] \eta[1] \eta[1, 1, p^2 + 1]!$$

(25)

The state of a massive spinless boson at rest is more complicated. Here the Para-Bose order has to be greater than one: $p^0 > 1$. The momenta are $P^0 = m$, $P^i = P^2 = P^3 = 0$.

$$\sum_{p^0 > 0} \sum_{p^2 > 0} \sum_{p^3 > 0} \frac{(-1)^{p^1 + p^2 + p^3} (m)^2 (p^1 + p^2 + p^3)!}{(2 p^1 + p^2 + p^3 + p^0 - 1)! p^0! p^0! p^0! (p^0 + 1 + p^2 + p^2 + p^3)! (p^0 + 1 + p^2 + p^2 + p^3)! (p^0 + 1 + p^2 + p^2 + p^3)! (p^0 + 1 + p^2 + p^2 + p^3)! (p^0 + 1 + p^2 + p^2 + p^3)! (p^0 + 1 + p^2 + p^2 + p^3)!}$$

(26)

$$\eta[4, 2, p^3] \eta[4, 1, p^2] \eta[3, 2, p^1] \eta[3, 1, p^1] \eta[3, 1, p^2] \eta[1, 1, p^2] \eta[1, 1, p^2]$$

These states, deriving from the irreducible representations of the Poincaré group, are constructed as eigenstates of the momenta, i.e., as plane waves. Via Fourier transformation the momentum representation can be converted into position representation. In relativistic quantum mechanics, the position representation raises some problems, but we forgo a further discussion here. A position applies to massive particles only, massless particles cannot be localized. However, even for massive particles problems arise if one attempts to localize the particle in a region smaller than its Compton wave length, as here one would encounter the creation of particle-antiparticle pairs. It should be noted that for massive particles the Newton-Wigner position operator is a suitable choice.[15]
7. How have interactions been treated in physics and which interactions are known?

Before turning to quantum-physical deliberations it may be useful to take once again a look at the interactions within the frame work of classical physics.

In classical mechanics, interacting particles are described in their respective position and velocity spaces. This may be not entirely obvious because of the fact that the individual spaces are assembled to form one common higher-dimensional phase-space.

Also in quantum mechanics, a position coordinate is assigned to every particle, which makes it possible that the identity of the particles is maintained under interactions. However, such a preservation of identity no longer applies to a qubit Fock representation of the particles – as shown above. Here, the tensor product of qubits appears as a quantum-physical whole, in which the individual identity of each interacting particle is lost.

For a description of interactions within the framework of the protyposis concept it will be necessary to allocate a particular position space to the manifold of the qubits associated with the particle under consideration and to separate this space from the space of qubits associated with the interacting remainder.

As his student and friend C. F. v. Weizsäcker sometimes recalled, W. Heisenberg used to characterize the position space as a parameter manifold of the interaction strength.

Which are the interactions and their characteristic strengths, physics presently knows?

One distinguishes four fundamental types of interactions, differing in the magnitude of their respective coupling constants. Three of them, gravitation, electromagnetism, and weak interaction, actually appear in space. The quanta of the latter can become manifest as real particles, or outgoing spherical waves.
7.1 Is it necessary to quantize gravitation, being a classical correction of a quantum cosmology?

The coupling constant attributed to gravitation is of the order $10^{-39}$. For the interaction of elementary objects, i.e., elementary particles, gravitation plays practically no role because of their small masses. However, for large masses, and, thus, in astrophysics, gravitation is the most important interaction. Gravitation is described very well by the equations of general relativity. This theory can be understood as a description of local inhomogeneities of the cosmic substance, i.e., of quantum fluctuations of the protyposis [16]. In a linearized form, general relativity allows one to describe the emission of gravitational waves from duple pulsars with extreme precision. A possible quantization of these linearized equations results in quanta representing massless gravitons with spin two. There have been many attempts, more or less successful, at a quantum-theoretical treatment of gravitation. However, these theories are not renormalizable, as here not only a few but infinitely many infinities would have to be set to zero. In order to devise a quantum field theory with spin-2-quanta, string theory postulates a tremendously inflated number of space dimensions – which very much looks like an act of desperation. To me, this makes apparent that "More of the Same" will not solve the existing problems, and a radical departure from the millennia old belief in “smallest particles as basis of reality” is unavoidable.

General relativity does not need a further quantization. It is a part of classical physics, proving to be a local correction of the protyposis quantum cosmology which itself invokes a thermodynamical postulate. The idea that general relativity can derived from thermodynamical postulates, thus, making a quantization questionable, is not new. To quote from a very remarkable paper by T. Jacobsen [17]:

“Viewed in this way, the Einstein equation is an equation of state. It is born in the thermodynamic limit as a relation between thermodynamic variables, and its validity is
seen to depend on the existence of local equilibrium conditions. This perspective suggests that it may be no more appropriate to quantize the Einstein equation than it would be to quantize the wave equation for sound in air.”

I do not think that attempts to quantize gravitation are dispensable. Eventually, solid state physics is not imaginable without phonons, quanta of sound, though they do not exist as free particles in the vacuum. Perhaps, also a quantization of gravitational waves may prove usefully in the future.

Einstein’s equations afford a very good approximation to reality, but they should not be seen as an axiomatic truth. This may be illustrated by the following consideration. There are infinitely many exact solutions of general relativity, of which a few are known. While each solution represents a whole universe with its full space-time evolution, at most one of them can be a description of the real cosmos, that is, the uni-versum, meaning the totality of all possible empirical and therefore scientific experience. All other solutions will have no relation to reality. This means, strictly speaking, Einstein’s equations claim too much, and, thus, it is hardly surprising that the attempts at quantization have not really been successful.

7.2 Is the strong interaction an interaction in space and time?

The strong interaction, with a coupling constant of the order of magnitude 1, is much stronger than the other interactions.

The strong interaction cannot be subsumed under of Heisenberg’s characterization of the position space as the parameter space of the interaction strength. The quanta of this force, the gluons, and the related fermions, the quarks, cannot be generated as free particles in space. The impossibility to isolate a quark can be illustrated with a simple analogy, namely, the attempt to cut off an “end” of a rubber band so that only the end is obtained without any piece of rubber attached. Of course, the end of a rubber band evidently exists, – though not as an independent entity. To isolate the end of a rubber band is as pointless as an attempt to isolate
a magnetic pole by breaking a bar magnet. At the breaking point, just a new pair of poles emerges. In the case of quarks, an additional quark-antiquark pair is generated when one tries to isolate a quark, that is, a meson is created rather than an isolated quark. Like a magnet, the meson can exist as an object in space and time. And just like magnet poles, quarks are essential quantum structures, which can generate measurable effects and are indispensable for an understanding of hadrons, irrespective of the fact that they cannot exist as objects. Of course, it is very useful to investigate interactions between quarks and gluons. However, while the strong interaction can be formulated as a gauge theory, one must not expect that this interaction is related in a simple way to the space-time continuum.

7.3 Quantum interactions in space and time

The coupling constant of the electromagnetic interaction, Sommerfeld’s fine structure constant $\alpha$, has a value of about $(1/137)$, the coupling constant of the weak interaction is of the order of $10^{-5}$. The corresponding quanta are, in the case of electromagnetic interaction, massless photons $\gamma$ having an infinite range, and the very massive $Z^0$ bosons together with the $W^+$ and $W^-$ bosons, being of very short range, in the case of weak interaction. These four quanta can exist in space and time as real particles.

While the strong interaction cannot apparently be described in the normal space-time continuum, the electromagnetic and the weak interaction seem to reflect the necessity of dividing reality into separated objects and correcting this separation by introducing interaction.

Let us examine this somewhat further.
8. How can interaction be modelled?

Usually, interaction is introduced into quantum theory in the framework of quantum field theory by designing a free Lagrange density and then replacing the original derivatives by covariant ones. The latter can be defined by means of gauge groups.

*This successful procedure cannot be used within the protyposis approach.*

Here, a reconstruction ansatz may provide arguments that will permit new insights in this matter. As has often been discussed, gauge theories also raise some questions, irrespective of their success. Experimental evidence shows that the respective gauge groups are well chosen, but there is still the issue of a deeper foundation of their mathematical structures. For instance, H. Lyre states [18]:

„Am Horizont sowohl der physikalischen als auch der philosophischen Untersuchungen über Eichtheorien deutet sich die noch völlig ungeklärte Frage nach einer noch tieferen Bedeutung der Konzeption der Eichtheorien an. Denn wenngleich das Eichprinzip – wie gezeigt – nicht zwingend auf nichtflache Konnektionen führt, so ist ja doch die in der kovarianten Ableitung vorgegebene Struktur des Wechselwirkungsterms auch für den empirisch bedeutsamen Fall nicht-verschwindender Feldstärken korrekt beschrieben. Diese *Wechselwirkungsstruktur* ist also tatsächlich aus der lokalen Eichsymmetrie-Forderung hergeleitet. Was aber ist der tiefere Grund für diese, zunächst rein formale Möglichkeit? Scheinbar handelt es sich um einen tiefliegenden und konzeptionell noch völlig unverstandenen Zusammenhang zwischen Raum und Wechselwirkung“

(At the horizon of both the physical as well as philosophical studies on gauge theories the still completely open question emerges of the deeper meaning of the conception of gauge theories. While the gauge principle – as shown – does not necessarily lead to non-flat connections, the structure of the interaction term, as determined by the covariant derivation, is correctly described even in the empirical significant case of non-vanishing field strength. This *structure of the interaction* is, in fact, derived from the requirement of
local gauge symmetry. But what is the deeper reason for this, so far only formal, possibility? Seemingly, there is a deeply rooted and conceptually completely unexplained relationship between space and interaction.

Starting from the protyposis it takes two quantization steps to reach the quantum fields. If one is looking for the deeper reasons underlying gauge theories and their groups, the simplest structures should be inspected rather than the most complex ones.

For a localized particle a possible interaction will be reflected by a change of the particle momentum. In case of a free particle, the momentum components are associated with the derivatives with respect the space-time coordinates, i.e., the generators of its translations in space and time:

$$P^i = -i \partial / \partial x_i$$  \hspace{1cm} (27)

How does the momentum change when a force acts on the particle?

### 8.1 Interaction requires separated objects

As indicated above, the description of interaction requires an amount of protyposis that is not affiliated with the position space of the particle under consideration. Otherwise the elementary qubit structure would not allow separating the total tensor product into interacting objects, but resulting in form of unitary quantum wholeness. However, the states of those protyposis qubits, which are not assigned to the considered object, can likewise be represented by functions on the pertinent symmetry groups, namely SU(2) and U(1), but here representing a distinct second space.

In the case of electromagnetic or weak interactions, where the position space parameterizes the interaction strength, the strength depends on the distance between the respective objects, i.e., their relative coordinates.
Accordingly, the theory of the protyposis suggests a description of interaction, in which one considers a combination of the generators of motion of the respective manifolds, i.e., the manifold of the particle and that of its interacting partner.

This means that the momenta of the considered object in Minkowski space have to be supplemented with the generators of motions of the group manifold that is not attributed to the particle. The parameter space of U(1) is one-dimensional, that of SU(2) three-dimensional. Let us denote the infinitesimal generators of these groups, elements of the Lie algebra, by $\mathbf{1}$ and $\mathbf{T}^a$.

For completeness, let us note that a group element being close to the unit element can be represented according to

$$ g = \exp\{\mathbf{i}A_1\} \approx 1 + \mathbf{i}A_1 \quad . $$

for U(1), and

$$ g = \exp\{\mathbf{i}\Sigma B_a \mathbf{T}^a\} \approx 1 + \mathbf{i}\Sigma B_a \mathbf{T}^a \quad . $$

for SU(2).

These relations apply to the motion on the group in the vicinity of the neutral element. This suggests the following substitution for the momentum components of the particle in Minkowski space:

$$ P^i \rightarrow -\mathbf{i}\partial/\partial x_i + g_1 A^i_1 + g_2 B^i_2 \mathbf{T}^a \quad . $$

Here $g_1$ und $g_2$ are couplings constants, the values of which are not yet determined by the foregoing analysis. According to the preceding considerations, the Minkowski space derivatives are transformed into covariant derivations, which, in the theory of local gauge
groups, are known for a long time. The gauge groups of the two interactions characterised by object-type interaction quanta, that is, quanta appearing as relativistic particles in space and time, can, in fact, be related to the space-time group of the protyposis.

The strong interaction, where the force and matter quanta do not exist as objects in space and time, points to a structure that has to be rationalized in a different way.

Acknowledgments

I thank Jochen Schirmer for many helpful hints.

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[9] see ref. [5]

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[15] A detailed description can be found e.g. in the work of Stefanovich, E. V.: (2012)
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[16] see ref. [11]


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